Clustering-based interference management in densely deployed femtocell networks

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1. Introduction

Wireless data traffic has been increasing dramatically, requiring more efficient use of the scarce radio spectrum. A significant fraction of the data traffic will come from indoor homes and offices. Because of the large cost to enhance the indoor coverage by adding macro base stations (MBSs), other solutions are being searched. Heterogeneous network, which consists of macrocells and the overlaying femtocells, is an economical and effective way to improve system capacity and coverage [1,2]. It complements and enhances existing macrocells by offloading mobile data traffic and saving radio/energy resources of macrocells [3]. Being an integrating part of future cellular networks, femtocells provide a new paradigm of network operation [4]. Particularly, plug-and-play femtocell base station (FBS) devices have been recently developed. Hence, femtocells can be owned privately and deployed randomly, which are opposed to well organized operators’ networks.

However, such a heterogeneous infrastructure also gives rise to nonnegligible challenges, which may seriously degrade the performance of the cellular networks [5]. Among all the challenges, resource allocation and interference management are most notable [6]. There are typically two types of resource-allocation schemes that account for macrocell and femtocell coexistence: shared spectrum [7,8] and split-spectrum schemes [9,10]. Wireless operators tend to favor co-channel deployment, where the FAPs and MBSs operate on the same licensed spectrum simultaneously in a universal frequency reuse fashion. This mode of operation has the benefit of high frequency reuse efficiency. However, in an orthogonal frequency-division multiple access (OFDMA)-based two-tier cellular network with spectrum sharing among femtocells and macrocells, the co-tier and cross-tier interferences significantly affect the network performance. In a two-tier cellular network, there are two kinds of interference [11]: cross-tier interference, that is, the aggressor (e.g., a femtocell user (FU)) and the victim of interference (e.g., a macrocell user (MU)) belong to different tiers; intra-tier interference, which means that the aggressor and the victim belong to the same tier. Hence, interference mitigation techniques need to be developed to manage the radio resources of femtocells in order to achieve the QoS requirements of all users.

The cross-tier interference can be mitigated by using suitable radio resource allocation methods (e.g., subchannel and power allocation methods) for the femtocells while the co-tier interference between...
neighboring femtocells can be mitigated by cooperative resource allocation among the femtocells (e.g., based on clustering of femtocells). Dense femtocell deployment is expected in the future [12], where the femtocells suffer from severe intra-tier interference due to dense deployment in a small area. Therefore, there are many new challenges that should be carefully addressed for the high density of femtocell scenario, such as resource allocation (RA) and interference management. Previous researches have provided an overview on interference avoidance mechanisms in a two-layer network [13], e.g., cell planning [14–16], power control [17,18], multiple antennas [19], adaptive femtocell access point (FAP) access scheme [20,21], and spectrum allocation [22–26]. These studies mainly focus on cross-tier interference mitigation. However, considering the fact that the number of FAPs is very large, many proposed intra-tier interference mitigation schemes are not scalable because they often yield a non-linear non-convex problem. Clustering can be used as a technique to reduce intra-tier interference by coordinating the transmissions of FAPs in a dense deployment scenario, which generally divides the RA task into a series of subproblems that are not difficult to deal with. The femtocells can be divided into disjoint clusters, where the entire set of subchannels is available for each cluster. However, no two femtocells in the same cluster are allowed to transmit on the same subchannel.

Hence, clustering-based interference mitigation schemes have been researched in the literature [27–30]. In [27], a clustering algorithm based on semi-definite programming is proposed to manage the intra-tier interference with a lower complexity. In [28], an efficient clustering algorithm is proposed to solve the interference management problem. However, it ignores the FU’s QoS requirements. A new game theoretic framework is proposed in [29], femtocell clustering is cast as an outer-loop evolutionary game coupled with bankruptcy channel allocation, which drives the cells to spontaneously switch to less interfered clusters. Within each cluster, it designs an inner-loop non-cooperative power control game, such that the requirement of prompt control is eliminated. In [30], a complete description of the interference in the form of its Laplace transform, the outage probability, coverage probability, and average achievable rate are derived in a K-tier HetNet where the BSs of each tier are randomly distributed by a clustered process. An important issue that follows is how to effectively assign orthogonal radio resources between macrocell and femtocells after dividing the femtocells into clusters meanwhile considering the cross-tier interference. In [31], the authors propose a dynamic clustering-based subband allocation scheme in a dense femtocell environment. In [32], a joint power control and resource allocation algorithm is developed for an orthogonal frequency division multiplexing (OFDM) femtocell net-

![Fig. 1. Network topology under consideration.](image)

work, where femtocells are grouped into disjoint clusters. In [33], cognitive radio technique is introduced to improve the performance of the femtocell networks.

In this paper, we formulate the clustering based subchannel and power allocation problem as an optimization problem. We try to maximize the sum throughput of all FUs while reducing the intra-tier interference and controlling the interference to the MU under its bearable threshold. Our general formulation leads to a computationally intractable problem, which is NP-hard. Therefore, it is divided into two procedures, the clustering and resource allocation. In the clustering, two femtocells which have strong interference with each other are grouped into clusters. And the femtocells in the same cluster use different subchannels to mitigate intra-tier interference. Then in each cluster, one femtocell is selected as the cluster center (CC) to perform subchannel and power allocation in this cluster. We propose a two-step method to address the resource allocation problem: subchannel allocation and power distribution. The subchannel allocation procedure can roughly satisfy the rate requirements of all FUs and the power allocation algorithm can achieve a near optimal solution. Numerical results validate the effectiveness and efficiency of our proposal.

The rest of this paper is organized as follows. In Section 2, we illustrate system model and formulate an optimization task. Section 3 discusses the clustering subproblem, together with the proposed low-complexity algorithm to obtain the best cluster configuration. In Section 4, we propose a suboptimal subchannel allocation algorithm and achieve an optimal power allocation scheme by developing an efficient fast method. Numerical results are given in Section 5 with discussions. Conclusion and future work are presented in Section 6.

### Table 1

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
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<tbody>
<tr>
<td>MBS</td>
<td>Macro base stations</td>
</tr>
<tr>
<td>FBS</td>
<td>Femtocell base station</td>
</tr>
<tr>
<td>MU</td>
<td>Macrocell user</td>
</tr>
<tr>
<td>FU</td>
<td>Femtocell user</td>
</tr>
<tr>
<td>FAP</td>
<td>Femtocell access point</td>
</tr>
<tr>
<td>CC</td>
<td>Cluster center</td>
</tr>
<tr>
<td>CM</td>
<td>Cluster member</td>
</tr>
<tr>
<td>N</td>
<td>Number of OFDM subchannel</td>
</tr>
<tr>
<td>Γ</td>
<td>SINR gap</td>
</tr>
<tr>
<td>C</td>
<td>Set of clusters</td>
</tr>
<tr>
<td>θ</td>
<td>Minimal cluster size</td>
</tr>
<tr>
<td>θ_c</td>
<td>Allowed maximal interference degree between two CCs</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
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<tbody>
<tr>
<td>kf</td>
<td>Femtocell i</td>
</tr>
<tr>
<td>f</td>
<td>Femtocell j</td>
</tr>
<tr>
<td>k</td>
<td>FU i</td>
</tr>
<tr>
<td>n</td>
<td>Subchannel</td>
</tr>
<tr>
<td>N</td>
<td>Number of clusters</td>
</tr>
<tr>
<td>R</td>
<td>Transmission rate</td>
</tr>
<tr>
<td>p</td>
<td>PSD of additive white Gaussian noise</td>
</tr>
<tr>
<td>w</td>
<td>Non-negative weight</td>
</tr>
<tr>
<td>θ_f</td>
<td>Subchannel set occupied by the k-th FU</td>
</tr>
</tbody>
</table>

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Part of this work has been presented at the IEEE ICC 2015, Shenzhen, China, November 2015 [34].
2. System model and problem formulation

Some frequently used notations are listed in Table 1.

Consider a two-tier heterogeneous network with densely deployed femtocells operating within a macrocell, as shown in Fig. 1. The femtocells are used to cover indoor area. In our study, we focus on the downlink communications based on OFDMA, whose frame structure can be viewed as time–frequency resource blocks. For simplification and convenience, we only consider the case of each femtocell with one FU. In such an environment, channels between FUes and their FAPs generally experience good propagation conditions. However, signals from outdoor macrocells are highly attenuated. Denote the set of femtocells by \( F \) with \( |F| = |f| \). We define that an FU belongs to femtocell \( f \) and the MU belongs to the macrocell is \( k_0 \). The bandwidth is divided into \( N \) OFDM subchannels in the cellular network.

Femtocell networks use cell-specific reference signals and unique cell-IDs. All FUs are capable of receiving the cell specific reference signals and identifying the interference source. In addition, femtocells are connected to the mobile core network, using the user’s broadband connection (digital subscriber line or cable television), via an intermediate entity called the FAP. The FAP can obtain all necessary information about channel gains between femtocells through FAP, based on which, the FAP can perform different clustering configurations. We assume that the signaling exchange between the FAP and femtocell is delay-free, since the FAP are interconnected within the cellular operator’s core network. Some of the N cells are connected to the FAP via cellular infrastructure (as highlighted by green link), whereas a larger number of cells are connected to the FAP through transport networks (e.g., edge networks).

Denote \( k_{f,n}^{c} \) by the channel gain between FU \( k_f \) and FAP \( n \) on the nth subchannel and we assume that perfect channel state information (CSI) is available at the transceivers of the MUs and the FUs.

In the indoor area where femtocells are densely deployed, the FU \( k_f \) and its serving FAP \( n \) are very close, so the channel gain between femtocell \( f \) and FU \( k_f \) is approximated to the channel gain between the two femtocells, i.e., \( k_{f,n}^{c} \approx k_{f,n}^{m} \) [35]. The \( k_f \)th FU has a minimal rate requirement of \( R_{k_f} \). The total available bandwidth of the system is \( W \). The interference to the MU introduced by FAP \( f \) on the nth subchannel with unit transmission power is \( I_{f,n}^{k_f} \).

Define the signal-to-interference plus noise ratio (SINR) of the \( k_f \)th FU in a macrocell on the nth subchannel as

\[
H_{k_f,n} = \frac{|k_{f,n}^{c}|^2}{\Gamma (N_0 W/N + I_{f,n}^{k_f})},
\]

where \( k_{f,n}^{c} \) is the channel gain of the \( k_f \)th FU over subchannel \( n \), \( N_0 \) is the PSD of additive white Gaussian noise, \( \Gamma \) is the SNR gap and can be represented as \( \Gamma = \frac{\ln(5)}{\ln(10)} \) for an uncoded multiple quadrature amplitude modulation (MQAM) with a specified bit error rate (BER). The interference caused by the MU’s signal is \( h_{k_f,n} \), which can be regarded as noise. And the transmission rate of the \( k_f \)th FU on the nth subchannel is

\[
r_{k_f,n} = \log_2(1 + p_{k_f,n} H_{k_f,n}).
\]

where \( p_{k_f,n} \) is the \( k_f \)th FU’s transmission power on the nth subchannel.

To reduce intra-tier interference, the femtocells can be divided into disjoint clusters. The idea behind clustering is to divide the joint subchannel and power allocation problem into smaller sub-problems. Denote the set of clusters as \( C \). A femtocell cluster \( c_m \subseteq C, \forall m \in \{1, 2, \ldots, |C|\} \), \( \bigcup_{m=1}^{M} c_m = C \), and \( \bigcap_{m=1}^{M} c_m = \emptyset \). Note that every cluster can use the entire set of subchannels \( N \) and no two femtocells in the same cluster transmit on the same subchannel in the meantime. In other words, there is no intra-tier interference within a cluster. As femtocells which have low interference with each other are grouped into different clusters, they can use the same subchannel for transmission. For very small cluster sizes, with one extreme being no clustering, the share of each femtocell in the available spectrum is high; however, the co-tier interference could be significant in this case. On the other hand, for large cluster sizes, co-tier interference among neighboring femtocells is minimized. However, the share of subchannels for each femtocell would be small. This suggests that cluster size is an important parameter to give a compromise between the share in the available spectrum and the co-tier interference.

Our target is to maximize the sum rate of the FUs under the transmit power limitation and the MU’s interference constraint while reducing the intra-tier interference, which leads to the following optimization problem:

\[
\max_{\text{s.t.}} \sum_{f=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{N} p_{k_f,n} r_{k_f,n}^{c_m} \geq R_{k_f}, \quad \forall k_f,
\]

\[
\max_{\text{s.t.}} \sum_{f=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k_f,n} r_{k_f,n}^{c_m} \geq R_{k_f}, \quad \forall k_f,
\]

\[
\sum_{f=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k_f,n} P_{k_f,n} \leq B, \quad \forall f,
\]

\[
\sum_{f=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k_f,n} P_{k_f,n} I_{f,n}^{k_f} \leq I_{k_f}, \quad \forall m,
\]

\[
\rho_{k_f,n} \geq 1, \quad \forall n, m,
\]

\[
\rho_{k_f,n} \geq 0, \quad \forall k_f, n.
\]

\[
\rho_{k_f,n} \in \{0, 1\}, \quad \forall k_f, n.
\]

where \( R_{k_f} \) is the minimal rate requirement of the \( k_f \)th FU. \( p_{k_f,n} \) can only be either 1 or 0, indicating whether the nth subchannel is used by the \( k_f \)th FU or not. \( P_t \) is the power limit of each femtocell and \( I_{k_f} \) is the interference power threshold of the MU. C1 is the throughput requirements of the FUs. C2 is the power limitation and C3 is the interference constraint, which enforces that the sum interference power at the MU in every cluster stays below \( I_{k_f} \). C4 is the exclusion constraint that in cluster \( c_m \), subchannel \( n \) can only be occupied by one femtocell. C5 and C6 indicate that the entire set of clusters \( C \) form the femtocell set \( F \) and the set of clusters are disjoint. C7 limits the maximum cluster size to \( S \). C8 and C9 are intuitive.

This problem is an MINLP whose solution is intractable. It includes both continuous and discrete variables. In addition, solving problem (2) requires a centralized mode of operation which is too complex for a practical solution. Hence, to solve this problem, we propose to divide it into two subproblems, i.e., the clustering sub-problem and the subchannel and power allocation sub-problem. First, the FAP gathers information about average channel gains among all the FAPs. The FAP sends this clustering information to the FAPs in every cluster stays below \( I_{k_f} \). C4 is the exclusion constraint that in cluster \( c_m \), subchannel \( n \) can only be occupied by one femtocell. C5 and C6 indicate that the entire set of clusters \( C \) form the femtocell set \( F \) and the set of clusters are disjoint. C7 limits the maximum cluster size to \( S \). C8 and C9 are intuitive.

3. Efficient clustering algorithm

Note that (3) defines a computationally intractable problem that involves variables \( c_m \), binary variables \( \rho_{k_f,n} \)'s and real variables \( p_{k_f,n} \)'s, which is NP-hard.
Table 2

Efficient clustering algorithm.

<table>
<thead>
<tr>
<th>Algorithm: Clustering algorithm for femtocells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Input: $\mathcal{W} = {W_1, W_2, ..., W_L}, z_1, ..., z_N, L, \theta_k, \theta_l, \theta_0$</td>
</tr>
<tr>
<td>2: While $l &lt; L$ and $\Delta T &gt; \theta_0$</td>
</tr>
<tr>
<td>3: Map femtocells into clusters</td>
</tr>
<tr>
<td>4: If $</td>
</tr>
<tr>
<td>5: Update new CCs according to (4)</td>
</tr>
<tr>
<td>6: For each $e_m$, update average interference degree $\Delta_{ij}$</td>
</tr>
<tr>
<td>7: Update average interference level for femtocell network $\mathcal{T}$</td>
</tr>
<tr>
<td>8: While $N_c &lt; E^2$</td>
</tr>
<tr>
<td>9: Calculate variance of each cluster $\sigma_m, \forall m$</td>
</tr>
<tr>
<td>10: Find $n^<em>_m$ satisfies $\sigma^</em>_m &gt; \sigma_m, \forall m$</td>
</tr>
<tr>
<td>11: $e_m$ splits into two clusters with CCs $z^<em>_m$ and $z^</em>_n$</td>
</tr>
<tr>
<td>12: End while</td>
</tr>
<tr>
<td>13: If there exist $z_i$ and $z_j, i \neq j$ that $w_{z_i} &gt; \theta_i$, combine cluster $e_i$ and cluster $e_j$</td>
</tr>
<tr>
<td>14: $l = l + 1$</td>
</tr>
<tr>
<td>15: End while</td>
</tr>
<tr>
<td>16: Return: Femtocell clusters $e_1, e_2, ..., e_N_c$</td>
</tr>
</tbody>
</table>

3.1. Optimal clustering

Optimal clustering can be obtained by an exhaustive search. For a given number of femtocells, all possible clustering configurations for the femtocells are tried. For a given clustering configuration, sub-channel and power allocation is performed. The cluster configuration yielding the highest sum-rate is the optimal cluster configuration. For F FAPs, the number of possible ways to cluster them is given by the Stirling Number of the Second Kind:

$$\sum_{m=1}^{L} \sum_{j=0}^{N} (-1)^{j+m} \approx O(2^L)$$  

(4)

It is clear that the number of possible cluster configurations (Bell Number) grows exponentially with the number of FAPs. Therefore, searching for the optimal cluster configuration by exhaustive search is prohibitive.

3.2. Efficient clustering algorithm

To reduce complexity and make the problem tractable, the original problem is divided into two sub-problems, the clustering and sub-channel and power allocation. In this section, we propose an efficient clustering scheme to reduce intra-tier interference among femtocells.

We propose an efficient clustering scheme to group the femtocells into clusters based on interference degree. Femtocells which have high interference degree with each other are grouped into the same cluster and in each cluster, no two femtocells transmit on the same subchannel. As femtocells which have low interference with each other are grouped into different clusters, they can use the same subchannel for transmission. In practice, femtocell density changes all times, so some clustering algorithms based on a given number of clusters are impractical. Our proposed scheme can change the cluster size and cluster number as the femtocell density varies, which is of practical merit.

To acquire the clustering formation, we model the femtocell network as an undirected graph $G = (V, E)$, where $V$ is the set of vertices which represents femtocells and $(i, j) \in E$ is the set of edges between two vertices. Every edge $(i,j)$ is given a non-negative weight $w_{i,j}$ which represents the interference degree between femtocell i and femtocell j. In the scene of femtocell networks, femtocell i and femtocell j have high $w_{i,j}$ if they have strong interference with each other. In fact, the two femtocells which have high channel gain $g_{i,j}$ between them will severely interfere with each other. Then, the weight $w_{i,j}$ is made in directly proportion to the channel gain between the two femtocells i, j by setting $w_{i,j} = g_{i,j}^\delta$.

The procedure is described in detail. The procedure initializes by setting up the femtocell interfering graph. Based on this graph, the femtocell gateway firstly selects arbitrary initial CCs, $z_1, z_2, ..., z_N_c$, where $N_c$ is the number of clusters. After all CCs are determined, the rest femtocells are then attached to the nearest CC and act as cluster members (CMs). A femtocell x belongs to the ith CC when $w_{z_i, x} > w_{z_j, x}, \forall j \neq i$, where $w_{x, y}$ is the interference degree between femtocell x and CC i while $w_{z_i, x}$ is the interference degree between femtocell x and CC j. We define $\mathcal{W}_f$ as the position of femtocell f in interference graph. When all femtocells are classified into clusters, we update the CCs by

$$\sigma_j = \frac{1}{1-L_j} \sum_{f \in C_j} \mathcal{W}_f, \quad j = 1, 2, ..., N_c$$  

(5)

Then the average interference degree $\sigma_j$ between CMs and CC in cluster $C_j$ is $\frac{1}{1-L_j} \sum_{f \in C_j} \mathcal{W}_f$, and the average interference degree of the femtocell network $\mathcal{T}$ is calculated by $\frac{1}{1-L} \sum_{j=1}^{N_c} \sigma_j$. If the number of femtocells in a cluster is less than our expected minimal cluster number, our clustering algorithm finds cluster with largest variance in which the interference level differs roughly and splits the cluster into two clusters. The number of clusters increases by one. And the variance in a cluster is calculated by $\sigma_j = \frac{1}{1-L_j} \sum_{f \in C_j} (w_{f, z_j} - \sigma_j)^2$. The splitting process continues until minimal cluster number is satisfied. Nevertheless, if the interference level between two CCs exceeds minimal interference level, which means the interference between the two clusters is large, then these two clusters should merge together. This process repeats until the stopping criteria is met.

4. Subchannel and power allocation

After getting the cluster configuration, the femtocell gateway sends these configurations in sequence to the femtocells through the wired backhaul. In each cluster, the CC will take charge with the subchannel and power allocation for all CMs in this cluster [27,28]. We try to maximize the sum capacity of all femtocells within each cluster, under minimal rate requirements for all FUs and the interference constraint for the MU. Therefore, we can formulate the RA problem in the cluster...
m for CC to solve:

\[
\max_{p_{k,f,n} \in [0,1]} \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} \delta t \quad C1: \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} \geq R_{k,f,min},
\]

\[
k_f = 1, \ldots, \lfloor k \rfloor.
\]

\[
k_f = 1 \quad \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} \leq P_f, \quad \forall \, f.
\]

\[
C3: \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} I_f^{1/n} \leq I_{th},
\]

\[
C4: \rho_{k,f,n} \geq 0, \quad \forall \, k, f, n.
\]

4.1. Suboptimal subchannel allocation

We propose a suboptimal approach to allocate subchannels to the FUs. In a femtocell network, the subchannel with high SNR for an FU may also bring more interference to the MU that uses this subchannel. In other words, the traditional water-filling-like method [36] is not appropriate because interference constraint also lays an upper bound of transmit power for each subchannel. That is to say, the interference introduced to the MU and the SINR of a subchannel should be jointly considered to calculate the rate of the subchannel. Our method measures the achievable rate of the nth subchannel used by the kth FU as follows,

\[
R_{k,f,n}^{max} = \log_2(1 + \rho_{k,f,n}^max H_{k,f,n}),
\]

where \(\rho_{k,f,n}^max\) is the maximum achievable power for the kth FU on the nth subchannel,

\[
p_{k,f,n} = \min(P_f, r_{k,f,n} I_f^{1/n}).
\]

Denote \(\Omega_k\) as the subchannel set occupied by the kth FU. We allocate the FUs subchannels to meet their minimal rate requirements. The principle of our subchannel allocation algorithm for the FUs is that the FU whose current rate is the farthest away from the target one has the priority to get a subchannel among the available ones. The procedure stops until all FUs' rate requirements are satisfied. For simplicity, the power of a subchannel is provisionally set as \(\min(P_f, \min_{f \in \Omega_k} (r_{k,f,n} I_f^{1/n}))\) to meet the power and interference limitations continuously. The operational procedure of the proposed algorithm for the cluster m is described in Table 3.

4.2. Fast barrier method for power allocation

After subchannel allocation, the power allocation problem in the cluster m can be rewritten as

\[
\max_{p_{k,f,n} \in [0,1]} \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} \delta t \quad C1: \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} \geq R_{k,f,min},
\]

\[
k_f = 1, \ldots, \lfloor k \rfloor.
\]

\[
k_f = 1 \quad \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} \leq P_f, \quad \forall \, f.
\]

\[
C3: \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} I_f^{1/n} \leq I_{th},
\]

\[
C4: \rho_{k,f,n} \geq 0, \quad \forall \, k, f, n.
\]

Eq. (9) defines a convex optimization problem and can be solved by barrier method [37]. Collect all \(p_{k,f,n}\)’s into one vector \(x\), the logarithmic barrier function is

\[
\phi(x) = -\sum_{k=1}^{K} \ln \left(\sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} - R_{k,f,min}\right) - \sum_{f=1}^{F} \ln \left(\sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n} I_f^{1/n}\right) - \sum_{k=1}^{K} \ln \rho_{k,f,n}.
\]

Note that the subscript \(k_f\) can be omitted as it has been determined by subchannel allocation. Denote

\[
f(x) = \sum_{k=1}^{K} R_{k,f}.
\]

where \(R_{k,f} = \sum_{f=1}^{F} \sum_{n=1}^{N} \rho_{k,f,n} r_{k,f,n}\), the optimal solution to (9) can be approximated by solving the following unconstrained minimization problem

\[
\min_{p_{k,f,n}} \phi(x) = -f(x) + \phi(x),
\]

where \(f \geq 0\) is a parameter to control the accuracy of solution. Newton method can efficiently solve this unconstrained minimization problem [37]. The Newton step at \(x\), denoted by \(\Delta x\), is given by

\[
\nabla \phi(x) \Delta x = -\nabla f(x),
\]

where \(\nabla f(x)\) and \(\nabla \phi(x)\) are the gradient and the Hessian of \(f(x)\), respectively. The procedure of the barrier method is outlined in Table 4.

The computational complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion. In order to reduce the computational cost, we exploit the structure of (9) and develop a fast algorithm to calculate the Newton step with lower complexity. Denote

\[
s_f = P_f - \sum_{n=1}^{N} \sum_{f=1}^{F} \rho_{k,f,n} r_{k,f,n} I_f^{1/n},
\]

\[
h_f = J_f - \sum_{n=1}^{N} \sum_{f=1}^{F} \rho_{k,f,n} r_{k,f,n} I_f^{1/n}.
\]

Table 4

<table>
<thead>
<tr>
<th>Barrier method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialization for Barrier method</td>
</tr>
<tr>
<td>Find feasible point (x): Set ((x^{(0)}) &gt; 0, \xi &gt; 0, \mu &gt; 1, 1)</td>
</tr>
<tr>
<td>Outer Loop for Barrier method</td>
</tr>
<tr>
<td>Stopping criterion of Barrier method: ((MKN + L)/\xi &lt; \xi)</td>
</tr>
<tr>
<td>Newton method</td>
</tr>
<tr>
<td>Tolerance (\xi &gt; 0)</td>
</tr>
<tr>
<td>Inner Loop for Newton method</td>
</tr>
<tr>
<td>Compute (\Delta x), and (\psi(x) = \psi(x^{(n)}))</td>
</tr>
<tr>
<td>Stopping criterion of Newton method: (</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Simulation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>System parameters</td>
</tr>
<tr>
<td>Radius of Macro-network: 500 m (LTE-A)</td>
</tr>
<tr>
<td>Radius of the femtocell: 20 m</td>
</tr>
<tr>
<td>Carrier frequency: 2 GHz</td>
</tr>
<tr>
<td>Total bandwidth: 10 MHz</td>
</tr>
<tr>
<td>Thermal noise PSD: (-174) dBm/Hz</td>
</tr>
<tr>
<td>Shadowing</td>
</tr>
<tr>
<td>Shadow fading: Log-normal</td>
</tr>
<tr>
<td>Macrocell parameters</td>
</tr>
<tr>
<td>Transmit power: 46 dBm</td>
</tr>
<tr>
<td>Antenna gain: 14 dBi</td>
</tr>
<tr>
<td>Noise figure: 7 dB</td>
</tr>
<tr>
<td>Femtocell parameters</td>
</tr>
<tr>
<td>Transmit power: 20 dBm</td>
</tr>
<tr>
<td>Noise figure: 7 dB</td>
</tr>
<tr>
<td>M(F)U parameters</td>
</tr>
<tr>
<td>Antenna gain: 0 dBi</td>
</tr>
<tr>
<td>Noise figure: 7 dB</td>
</tr>
</tbody>
</table>
The Hessian of $\psi(x)$ is

$$\nabla^2 \psi(x) = \begin{bmatrix} D_1 & D_2 & \cdots & D_N \end{bmatrix} + \sum_{f=1}^{M} \sum_{j=1}^{N} \frac{\nabla f_j \nabla f_j^T}{s_f} + \sum_{k_f=1}^{N} \frac{\nabla f_k \nabla f_k^T}{f_{k_f}}$$

(15)

where $D = \text{diag}(D_1, D_2, \ldots, D_N)$ and $M = 2|v_i| + 1$ with

$$D_n \left( t + \frac{1}{R_{k,f}} \left( \frac{1}{1 + p_{k,f}H_{k,f}} \right)^2 + \frac{1}{p_{k,f}} \right).$$

(16)

$F_i$ are all vectors with $N$ elements,

$$F_i = \begin{cases} \frac{\nabla f_i}{s_f}, & f = 1, \ldots, |v_i|, \quad i = f, \\ \frac{\nabla f_{k_f}}{f_{k_f}}, & k_f = 1, \ldots, |v_i|, \quad i = k_f + |v_i|, \\ \frac{\nabla f_{k_f}}{s_o}, & i = 2|v_i| + 1. \end{cases}$$

(17)

**Theorem 1.** The problem defined in (9) can be solved with the complexity of $O(M^2N)$.
We give the proof in detail in Appendix. If we solve (9) via standard convex optimization technique, it has a complexity of $O(N^3)$. In practical wireless systems, $M \ll N$ and our proposed algorithm has a significant advantage to solve the RA problem that can be tackled in an online manner.

5. Numerical results and discussions

Consider an LTE-advanced network where a macrocell is in the center of a circle with radius of 500 m. Each FU is uniformly distributed within a circle with radius of 20 m from the pairing FAP. We consider an indoor area with dense deployed femtocells within the coverage of the macrocell. A dual-stripe building model, which was initially proposed in [38], is adopted to evaluate the performance of our algorithm. The simulation parameters are listed in Table 5.

The distance dependent path loss attenuation varies according to the characteristics of the evaluated link. We give a summary of the different situations in our simulations.

- **Macroseell to MU:**
  \[ PL(d) = 15.3 + 37.6 \log(d) + L_{\text{ew}}, \]
  where $d$ (in m) is the distance between the macrocell to the indoor MU/FU and $L_{\text{ew}}$ is the penetration loss in the external walls of the building.

- **Femtocell to FU:**
  \[ PL(d) = 38.46 + 20 \log(d) + 0.7d_{2D} + qL_{\text{ew}} + 18.3n^{0.25 + 0.46}, \]  \hspace{1cm} (18)
  where $d$ is the distance between the femtocell to the FU, $d_{2D}$ is the indoor distance of the link, $L_{\text{ew}}$ is the penetration loss in the internal walls of the building, $q(n)$ denotes the number of penetrated walls (floors).

Shadow fading is modeled as a log-normal random variable, whose standard deviation is 4 dB and 8 dB for the MU and the FUs respectively. About fast fading, in the frequency domain, the channel gains for subchannels are modeled as independent and identically distributed zero-mean circularly symmetric complex Gaussian random variables.

The parameters of the clustering algorithm are as follows: the maximal iterations $L$ for clustering is set by 20 and the maximal interference degree between two CCs is $10^{-16}$ W.

Fig. 2 shows the average capacity of femtocells as a function of power limit achieved by our proposed algorithm with other two algorithms: equal power allocation (EPA) algorithm and IFPA [39] based on the same clustering and subchannel allocation methods proposed above. EPA assumes that power is equally allocated among all subchannels and IFPA allocates power inversely proportional to the interference level. From Fig. 2 we can see that the average capacity of the FUs grows with the increase of the power budget. Our proposed algorithm performs better than the other algorithms. When power budget grows larger, our algorithm performs much better than the EPA and IFPA.

Fig. 3 shows the variation in femtocell data rate with the macrocell power. The average data rate achieved with clustering using the Kmeans technique and Similarity Clustering are shown as well. Similarity Clustering is introduced in [28]. We observe that our clustering has a performance that is close to the optimal solution and better than the Similarity Clustering and Kmeans approach. Fig. 3 shows that as the macrocell power increases, the cross-tier interference increases and hence, the achieved data rate decreases. Although the cross-tier interference becomes more dominant, clustering is still beneficial.

Fig. 4 shows the variation in femtocell data rate with the interference threshold. We have $P_{\text{max}}=30$ mW, $P_{\text{macro}}=20$ W, $L_{\text{low}}=30$ dB, and $q_{\text{Liw}}=5$ dB. The average data rate achieved with clustering using the Kmeans technique and Similarity Clustering are shown as well. Similarity Clustering is introduced in [28]. We observe that our clustering has a performance that is close to the optimal solution and better than the Similarity Clustering and the Kmeans approach. Correlation clustering reduces the search space for the optimal cluster configuration with the drawback of the possibility of missing the optimal cluster configuration. It is observed that the performance of this scheme can be even worse than that of the uncoordinated scheme.

We also study the average capacity of femtocell networks in various femtocell densities in Fig. 5. We compare the performance of our proposed clustering algorithm with $K$-means algorithm. $K$-means algorithm is introduced in [40], which executes clustering based on a given cluster size and cluster number. Both of the two algorithms have a complexity of $O(K_F)$, which $K_F$ is the number of all FUs. Both algorithms decrease as the femtocells density increases. However, the capacity in our proposed algorithm is higher than the $K$-means algorithm. This is mainly because that in the $K$-means algorithm the cluster size and the number of clusters are predefined, which is not fit for different femtocell intensively. Our algorithm dynamically changes the cluster size and number of clusters as the femtocell density changes.

Finally, we investigate the convergence of our proposed fast algorithm. As discussed above, the computational load of the proposed algorithm mainly lies in the computation of Newton step. Fig. 6 shows the number of Newton iterations for the barrier method to converge in 100 random instances. Fig. 7 gives the cumulative distribution function (CDF) of the number of Newton iterations for solving the optimal power allocation with different number of $N$. As seen in Fig. 7, the number of Newton iterations is not large and varies in a narrow range, indicating that our proposed algorithm is efficient.

6. Conclusion

In this paper, we studied the RA and interference management problem in dense OFDM femtocell networks. In this context, the FAP will be responsible for the clustering phase, and then the CH (elected from the femtocell group) will be responsible for the sub-channel and power allocation phase. Our formulation leads to a mixed integer programming problem which is computationally intractable. So we divided the problem into two subproblems: clustering subproblem and subchannel and power allocation subproblem. First, femtocells are grouped into clusters to lower intra-tier interference. Then, the CCs will be responsible for the sub-channel and power allocation in each cluster. We allocate subchannels to FUs by considering the rate gap between each FU’s current rate and its requirement. Finally, we develop a fast algorithm which can achieve the optimal power distribution with a


Rewrite the KKT system (13) as follows,

\[ A_0 \Delta x = F_0, \]  

where \( A_0 = \nabla^2 \psi \) and \( F_0 = -\nabla \psi \). According to the (15), \( A_0 \) can be written as

\[ A_0 = D + \sum_{i=1}^{M} F_i F_i^T, \]  

which can be decomposed into \( M \) equations,

\[ A_i = A_{i1} + F_i F_i^T, \quad i = 0, 1, \ldots, M - 1. \]  

By exploiting the structure of \( A_i \)'s, we give an \( M \)-step procedure to compute Newton step efficiently.

First, use (A.3) to decompose \( A_0 \) that is, \( A_0 = A_1 + F_1 F_1^T \). Denote two intermediate variables as the solutions of the following linear equations:

\[ A_1 v_1 = F_1, \]  

\[ 1. \]  

which can be obtained by\( A_1 \) with \( A_1 = A_2 + F_2 F_2^T \). Then the two variables introduced in step 1 can be updated by solving the following three sets of linear equations, \( A_2 v_2 = F_2, i = 1, 2, 3 \), where \( v_1, v_2 \) and \( v_3 \) are three new intermediate variables.

For the \( m \)-th step, decompose \( A_{m-1} \) with \( A_0 = A_m + F_m F_m^T \). We can update the \( m \) variables introduced in step \( m \) by \( v_m = v_{m-1} - \frac{F_m v_{m-1}^T}{1 + F_m v_{m-1}^T} v_{m-1} \) and \( v_m \). Continue the procedure, decompose \( A_{m-1} \) with \( A_{m-1} = A_m + F_m F_m^T \). Then the two variables introduced in step 1 can be updated by solving the following \( (m+1) \) sets of linear equations, \( A_m v_m = F_m, i = 1, 2, \ldots, M + 1 \). From the derivation process, we can find that the \( m \) variables \( v_{m-1}, i = 1, 2, \ldots, m \) in the \( (m-1) \) th step can be obtained by solving the \( m+1 \) variables \( v_m, i = 1, 2, \ldots, m + 1 \). Since we figure out the \( m+1 \) variables \( v_m, i = 1, 2, \ldots, M + 1 \), \( \Delta x \) will be indirectly obtained.

Equation \( A_M v_M = F_1 \) can be solved as follows: According to the analysis given in Section 4, we have \( A_M = D \). Unify these equations into

\[
\begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_N
\end{bmatrix} = \begin{bmatrix} 1 \\
\vdots \\
1 \end{bmatrix} g,
\]

Since \( D \) is a diagonal matrix, we can easily obtained

\[ v_i = D_i^{-1} g_i, \quad i = 1, \ldots, N. \]

Thus the computational complexity of solving the \( M+1 \) matrix systems is \( O(MN) \). We also need an \( M \)-step reverse iteration to figure out \( \Delta x \). The total computation cost of the proposed method is \( O(MN) \).

References


